FREE ACTIONS OF GROUPS ON HOMOTOPY SPHERES

JANG HYUN JO

ABSTRACT. We study the problems concerning on free actions of groups on a space which is homotopy equivalent to a sphere or a product of spheres. The question that the periodicity of group cohomology is the algebraic characterization of those groups G which admit a finite dimensional free G-CW-complex which is homotopy equivalent to a sphere has long been conjectured to be true. One of the purposes of this manuscript is to present a new conjecture which implies the above conjecture and provide a class of groups satisfying it. We also consider another conjecture of longstanding interest in algebraic topology which is the extension of a single sphere to the case of a product of spheres as follows: If $F = (\mathbb{Z}_p)^r$ acts freely on a finite dimensional CW-complex X which is homotopy equivalent to a product of spheres $S^{n_1} \times \cdots \times S^{n_k}$, then $r \leq k$. Another purpose of this manuscript is to say that it is sufficient for proving this conjecture to consider only the cases $n_i \geq 2$.

1. INTRODUCTION

Let G be a finite group which admits a finite (dimensional) free G-CW-complex X homotopy equivalent to a sphere. It is easy to show that G has periodic cohomology (after 0-step), i.e., $H^i(G)$ and $H^{i+n+1}(G)$ are naturally equivalent for all $i > 0$. In [24], Swan showed that the converse of this is also true, i.e., if a finite group G has periodic cohomology, then it acts freely on a finite complex which is homotopy equivalent to a sphere. Similarly, if an infinite group G acts freely on a finite-dimensional complex which is homotopy equivalent to a sphere, then it can be showed that G has periodic cohomolgoy after k-steps. In [22, 25] , Mislin and Talelli have conjectured that whether the notion of the periodic cohomology after k-steps is an algebraic mirror to the phenomenon of a free action on a finite dimensional homotopy sphere as follows:

Conjecture A. A group G has periodic cohomology after some steps if and only if G admits a finite-dimensional free G-CW-complex which is homotopy equivalent to a sphere.

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For more details, we recommend [2, 22, 25]. Mislin and Talelli [22] proved Conjecture A when the group G belongs to the class $\mathbf{H}\mathfrak{F}_b$ of groups (see Section 2 for the definition of the class $\mathbf{H}\mathfrak{F}_b$). Adem and Smith [2] proved Conjecture A under the hypothesis that the periodicity isomorphisms given by the cup product with a cohomology element. In [25], Talelli combined the following with the result of Adem and Smith to show that Conjecture A holds for the groups of the class $\mathbf{H}\mathfrak{F}$.

Theorem A. ([25, Theorem 3.2, Corollary 3.3, Proposition 3.4]). Let G be a group with periodic cohomology of period q after k-steps. Then the following statements are equivalent:

(1) $H^i(G, P) = 0$ for all $i > k$ and every projective $\mathbb{Z}G$-module P.
(2) The periodicity isomorphism are induced by the cup product with an element in $H^q(G, \mathbb{Z})$.
(3) $\operatorname{spli} G < \infty$.

Moreover, for $G \in \mathbf{H}\mathfrak{F}$ the above equivalent conditions hold. Thus if $G \in \mathbf{H}\mathfrak{F}$, then Conjecture A holds for G.

Notice that the condition (1) of Theorem A is equivalent to the condition $\operatorname{pccd} G \leq k$ (cf. [12, Proposition 2.3]). It was also known from [12, Theorem 3.17] that if $G \in \mathbf{H}\mathfrak{F}$ or $\operatorname{pccd} G < \infty$, then the condition (1) of Theorem A holds. Thus if G has periodic cohomology of period q after k-steps, then every proper subgroup $H < G$ of finite projective complete cohomological dimension satisfy $\operatorname{pccd} H \leq k$, since H has also periodic cohomology of period q after k-steps. In this view point, we may have the following conjecture whose analogous question was also considered in [15, 23]:

Conjecture B. Suppose that there exists a nonnegative integer k such that for each proper subgroup $H < G$ of finite projective complete cohomological dimension, $\operatorname{pccd} H \leq k$. Then $\operatorname{pccd} G < \infty$.

As noted above, if Conjecture B is a theorem then so is Conjecture A. The first aim of this manuscript is to give a large class of groups which satisfy Conjecture B and therefore Conjecture A as follows:

Theorem B. The groups of the class $\mathfrak{X}$ satisfy Conjecture B. Furthermore, the groups of the class $\mathbf{L}\mathfrak{X}$ satisfying the $\aleph_n$-condition satisfy Conjecture B.

On the other hand, there is a conjecture of longstanding interest in Algebraic topology which is the extension of a single sphere to the case of a product of spheres.

Conjecture C. If $F = (\mathbb{Z}_p)^r$ acts freely on a finite dimensional CW-complex X which is homotopy equivalent to a product of spheres $S^{n_1} \times \cdots \times S^{n_k}$, then $r \leq k$.

This was solved for products of two spheres by Heller. And it was settled in the equidimensional case in work by Carlsson [7], Browder [5], and Benson-Carlson [3] under the assumption of trivial action on homology. The homologically nontrivial case was answered by Adem-Browder in [1], except when $p = 2$ and the dimension of the spheres is $1, 3$, or 7. In [26], Yalçin solved the case of $(S^1)^k$. For other cases, it is still open.

The other aim of this manuscript is to generalize Yalçin's result [26, Corollary 5.2] from tori to compact solvmanifolds and give the following interesting result related to Conjecture C:

Theorem C. Suppose that Conjecture C is true for the cases that $n_i \geq 2$. Then Conjecture C is a theorem.

2. Preliminaries

In this section, we briefly recall the necessary background information that will be used in main results. For more details, see corresponding references.

1. The classes of groups $\mathbf{H}\mathfrak{F}$ and $\mathbf{LH}\mathfrak{F}$ was introduced by Kropholler [17]. Let $\mathbf{H}_0\mathfrak{F}$ be the class of finite groups. Define $\mathbf{H}_\alpha\mathfrak{F}$ for each ordinal α inductively: if α is a successor ordinal, then $\mathbf{H}_\alpha\mathfrak{F}$ is the class of groups G which admit a finite dimensional contractible G-CW-complex with isotropy groups in $\mathbf{H}_{\alpha-1}\mathfrak{F}$, and if α is a limit ordinal, then $\mathbf{H}_\alpha\mathfrak{F} = \cup_{\beta<\alpha}\mathbf{H}_\beta\mathfrak{F}$. A group belongs to $\mathbf{H}\mathfrak{F}$ if it belongs to $\mathbf{H}_\alpha\mathfrak{F}$ for some α and belongs to $\mathbf{LH}\mathfrak{F}$ if all of its finitely generated subgroups belong to $\mathbf{H}\mathfrak{F}$. The class $\mathbf{H}\mathfrak{F}_b$ is defined by the subclass consisting of those groups in $\mathbf{H}\mathfrak{F}$ for which there is a bound on the orders of their finite subgroups [22]. It was well-known that the class $\mathbf{LH}\mathfrak{F}$ contains all elementary amenable groups, all linear groups, and all groups of finite virtual cohomological dimension. And it is subgroup closed, extension closed, and closed under arbitrary directed unions.

2. Let G be a discrete group. The complete cohomology of G was introduced independently by Benson and Carlson [4], Mislin [21] and Vogel [9] and their approaches turned to be all isomorphic by Mislin. The adventure of complete cohomology naturally gave rise to the notion of projective complete cohomological dimension $\operatorname{pccd} G$ of a group G [12]. This invariant is defined as the least integer $n \geq -1$ for which $H^i(G, -) \cong \widehat{H}^i(G, -)$ for all $i > n$, or ∞ if no such n exists. Here we denote the complete cohomology of G by $\widehat{H}^i(G, -)$. It was conjectured that $\operatorname{pccd} G = \infty$ can not occur [12, 16]. This conjecture was proved to be true for many classes of groups and some other related conjectures was investigated in [12]. However, it turned out from anonymous mathematician that there is a counter-example to this conjecture. For example, if $G = *_{n\in\mathbb{N}}G_n$ and G_n is a free abelian group of rank n, then $\operatorname{pccd} G = \infty$. This follows immediately from the cohomology version of the Mayer-Vietoris sequence (cf. [6]). But, this is a still conjecture when a group G has periodic cohomology.

3. Let $B(G,\mathbb{Z})$ be the $\mathbb{Z}G$-module of functions from G to $\mathbb{Z}$ which takes only finitely many different values in $\mathbb{Z}$. Notice that $B = B(G,\mathbb{Z})$ is projective over $\mathbb{Z}H$ for any finite subgroup H of G. Thus if a group G admits a finite dimensional universal proper G-CW-complex $\underline{E}G$, then $\text{proj.dim}_{\mathbb{Z}G} B(G,\mathbb{Z}) < \infty$ by [20, Lemma 1.5]. Also if G is an $\mathbf{H}\mathfrak{F}$-group of type FP_∞ then $\text{proj.dim}_{\mathbb{Z}G} B(G,\mathbb{Z}) < \infty$ [18, Theorem A]. It was known from [12] that if $\text{proj.dim}_{\mathbb{Z}G} B(G,\mathbb{Z}) < \infty$, then $\text{pccd}\, G < \infty$.

4. The cohomological dimension of G, denoted $\text{cd}\, G$, is the projective dimension of the trivial G-module $\mathbb{Z}$ over $\mathbb{Z}G$. For a virtually torsion-free group G, i.e., G has a torsion-free subgroup of finite index, it was well-known that all torsion-free subgroups of G of finite index have the same cohomological dimension (cf. [6]). The common cohomological dimension of the torsion-free subgroups of finite index is called the virtual cohomological dimension of G and is denoted by $\text{vcd}\, G$. The finiteness of $\text{vcd}\, G$ ensures that the Farrell cohomology of a group is well defined. There are the following well-known invariants of a group which have been accompanied with the Ikenaga's generalized cohomology and the complete cohomology of a group:

(1) $\underline{\text{cd}}\, G := \sup\{\, n \,:\, \text{Ext}^n_{\mathbb{Z}G}(M,F) \neq 0,\ M : \mathbb{Z}\text{-free}\}$ [10].
(2) $\text{spli}\, G := \sup\{n \,:\, \text{Ext}^n_{\mathbb{Z}G}(I,-) \neq 0,\ I : \mathbb{Z}G\text{-injective}\}$ [8].
(3) $\text{silp}\, G := \sup\{n \,:\, \text{Ext}^n_{\mathbb{Z}G}(-,P) \neq 0,\ P : \mathbb{Z}G\text{-projective}\}$ [8].
(4) $\text{pccd}\, G := \inf\{n \,:\, H^i(G,-) \cong \widehat{H}^i(G,-),\ i > n\}$ [12].

It was well known that for any group G,

$$\text{pccd}\, G \leq \underline{\text{cd}}\, G \leq \text{silp}\, G \leq \underline{\text{cd}}\, G + 1 \ \text{([8, 10, 12])}.$$

The following results show how $\text{pccd}\, G$ behaves with respect to extensions and amalgamations [12]:

(1) If $1 \to N \to G \to Q \to 1$ is a exact sequence of groups, then $\text{pccd}\, G \leq \text{pccd}\, N + \text{vcd}\, Q$;
(2) If $G = A *_C B$, then $\text{pccd}\, G \leq \max\{\text{pccd}\, A, \text{pccd}\, B, 1 + \text{pccd}\, C\}$.

5. Let S be a connected and simply connected solvable Lie group and H be a closed subgroup of S. The coset space $H\backslash S$ is called a solvmanifold. Let $\pi = \pi_1(M)$ be the fundamental group of the solvmanifold $M = H\backslash S$. Then $\pi = H/H_0$, where H_0 is the identity component of H. Such a group is known to be a Mostow-Wang group or a strongly torsion-free solvable group, i.e., π contains a finitely generated torsion-free, nilpotent normal subgroup Λ with the quotient π/Λ free abelian of finite rank [19]. That is, π fits into the following exact sequence

$$1 \longrightarrow \Lambda \longrightarrow \pi \longrightarrow \mathbb{Z}^n \longrightarrow 1.$$

The Hirsch length of a strongly torsion-free solvable group is well defined, which is the sum $h(\Lambda) + n$.

3. Proof of Results

We start with the following lemma.

Lemma 3.1. *Let G admit a n-dimensional contractible G-CW-complex X. If there exists a nonnegative integer k such that for any isotropy subgroup G_σ, $\operatorname{pccd} G_\sigma \leq k$, then $\operatorname{pccd} G \leq n+k$.*

Proof. It follows from the following spectral sequence (cf. [6])

$$E_1^{p,q} = \prod_{\sigma \in \sum_p} H^q(G_\sigma, M) \Rightarrow H^{p+q}(G, M),$$

where M is a $\mathbb{Z}G$-module and $\sum_p$ is a set of representatives for the p-simplices of X mod G. □

Lemma 3.2. *Let $G = \varinjlim_{i \in I} G_i$, where $G_i < G$ and $|I| = \aleph_n$. Then*

$$\operatorname{pccd} G \leq \sup_{i \in I}\{\operatorname{pccd} G_i\} + n + 1.$$

Proof. It can be proved by the same method of [10, Proposition 6]. □

Proposition 3.3. *Let $G = \bigoplus_{i \in I} G_i$ be a direct sums of groups G_i with $|I| = \aleph_n$. If each G_i satisfies Conjecture B, then so does G.*

Proof. Suppose that there exists a nonnegative integer k such that for each proper subgroup $H < G$ of finite projective complete cohomological dimension, $\operatorname{pccd} H \leq k$. Then for each $i \in I$ and for any proper subgroup $H_i < G_i$ of finite projective complete cohomological dimension, $\operatorname{pccd} H_i \leq k$. Since G_i satisfies Conjecture B, we have $\operatorname{pccd} G_i < \infty$ and thereby $\operatorname{pccd} G_i \leq k$ for each i. Hence $\operatorname{pccd} G < \infty$ by Lemma 3.2. □

Proposition 3.4. *Let $1 \to N \to G \to Q \to 1$ be an extension of groups such that $\operatorname{vcd} Q < \infty$. If N satisfies Conjecture B, then so does G.*

Proof. Suppose that there exists a nonnegative integer k such that for each proper subgroup $H < G$ of finite projective complete cohomological dimension, $\operatorname{pccd} H \leq k$. Notice that for each proper subgroup $L < N$ of finite projective complete cohomological dimension, $\operatorname{pccd} L \leq k$. Since N satisfies Conjecture B, $\operatorname{pccd} N < \infty$ and thereby $\operatorname{pccd} N \leq k$. Since $\operatorname{pccd} G \leq \operatorname{pccd} N + \operatorname{vcd} Q$, we have $\operatorname{pccd} G < \infty$. □

Proposition 3.5. *Suppose that G admits a finite-dimensional contractible G-CW-complex X. If each isotropy group G_σ of X satisfies Conjecture B, then so does G.*

Proof. Suppose that there exists a nonnegative integer k such that for each proper subgroup $H < G$ of finite projective complete cohomological dimension, $\operatorname{pccd} H \leq k$. Notice that for each isotropy group G_σ, any subgroup $H_\sigma < G_\sigma$ of finite projective complete cohomological dimension satisfies $\operatorname{pccd} H_\sigma \leq k$. Since G_σ satisfy Conjecture B, $\operatorname{pccd} G_\sigma < \infty$ and thereby $\operatorname{pccd} G_\sigma \leq k$. Hence $\operatorname{pccd} G < \infty$ by Lemma 3.1. □

Using Proposition 3.5 and the transfinite induction, we have the following corollary.

Corollary 3.6. *Let G be a group of type* $\mathbf{H}\mathfrak{F}$*. Then Conjecture B holds for G. If, in addition, G is an* $\mathbf{LH}\mathfrak{F}$*-group satisfying the $\aleph_n$-condition, then Conjecture B holds for G.*

Definition 3.7. Let $\mathfrak{X}$ denote the smallest class of groups which

(1) contains all groups of type $\mathbf{H}\mathfrak{F}$
(2) contains all groups G with $\operatorname{silp} G < \infty$
(3) contains all groups G with $\operatorname{proj.dim}_{\mathbb{Z}G} B(G, \mathbb{Z}) < \infty$
(4) is closed under direct sums of groups $\bigoplus_{i \in I} G_i$ with $|I| = \aleph_n$
(5) is closed under extensions of groups $1 \to N \to G \to Q \to 1$ such that $\operatorname{vcd} Q < \infty$
(6) is closed under passing to the group G which admits a finite-dimensional contractible G-CW-complex from isotropy groups G_σ.

A group belongs to $\mathbf{L}\mathfrak{X}$ if all of its finitely generated subgroups belong to $\mathfrak{X}$.

Theorem 3.8. *(= Theorem B) The groups of the class $\mathfrak{X}$ satisfy Conjecture B. Furthermore, the groups of the class $\mathbf{L}\mathfrak{X}$ satisfying the $\aleph_n$-condition satisfy Conjecture B.*

Proof. If G belongs to $\mathcal{X}$, then the result follows from Lemma 3.2, Propositions 3.3,3.4,3.5, Corollary 3.6 and our preliminaries in Section 2. If G is an $\mathbf{LH}\mathfrak{F}$-group satisfying the $\aleph_n$-condition, then the result follows from Lemma 3.2. □

Corollary 3.9. *The groups of the class $\mathfrak{X}$ satisfy Conjecture A. Furthermore, the groups of the class $\mathbf{L}\mathfrak{X}$ satisfying the $\aleph_n$-condition satisfy Conjecture A.*

Proof. Recall that if Conjecture B is a theorem then so is Conjecture A. □

Let p denote a prime number. By the p-rank of a finite group F, denoted by $\operatorname{rk}_p(F)$, we mean the largest rank among elementary abelian p-subgroups of F. Let G be a virtually torsion-free group. Then there is a normal torsion-free subgroup N in G so that the quotient group G/N is finite. Every finite subgroup of G is isomorphic to a subgroup of G/N. Thus there are finitely many isomorphism classes for the finite subgroups of G. Hence it makes sense to define the p-rank of G, denoted $\mathrm{rk}_p(G)$, to be the largest rank among elementary abelian p-subgroups of G.

Now consider the following theorem which is a crucial ingredient for our second main result. This is a generalization of Yalçin's result [26, Theorem 3.2] and its proof is appeared in [14].

Theorem 3.10. *Let Γ be a strongly torsion-free solvable group and F be a finite group. Let $1 \to \Gamma \to \Pi \to F \to 1$ be an extension. Then $\mathrm{rk}_p(F) - \mathrm{rk}_p(\Pi) \leq h(\Gamma)$ for all prime p with $p \mid |F|$.*

Corollary 3.11. *Let F be a finite group which acts freely on a compact solvmanifold M. Then $\mathrm{rk}_p(F) \leq \dim(M)$ for all prime p.*

Proof. Consider the exact sequence $1 \to \pi_1(M) \to \pi_1(F\backslash M) \to F \to 1$, where $\pi_1(M)$ is a strongly torsion-free solvable group whose Hirsch length equals the dimension of M. The result follows immediately from the above theorem. □

Corollary 3.12. *Suppose that a finite group F acts freely on the product of a compact solvmanifold M and a space L with finite fundamental group. Then:*

(1) *$\pi_1(M \times_F L)/\pi_1(M)$ fits into the extension*

$$1 \to \pi_1(L) \to \pi_1(M \times_F L)/\pi_1(M) \to F \to 1.$$

(2) *For all prime p with $p \mid |\pi_1(M \times_F L)/\pi_1(M)|$,*

$$\mathrm{rk}_p(\pi_1(M \times_F L)/\pi_1(M)) - \mathrm{rk}_p(\pi_1(M \times_F L)) \leq \dim(M).$$

Proof. The fundamental group of the orbit space $M \times_F L$ fits into the following exact sequence $1 \to \pi_1(M \times L) \to \pi_1(M \times_F L) \to F \to 1$. Note that $\pi_1(M \times L) = \pi_1(M) \times \pi_1(L)$ and $\pi_1(M)$ is a strongly torsion-free solvable group whose Hirsch length equals the dimension of M. Now our result follows from the theorem above. □

Corollary 3.13. *(= Theorem C) Suppose that the conjecture above is true for the cases that $n_i \geq 2$. Then the conjecture above is a theorem.*

Proof. Let $F = (\mathbb{Z}_p)^r$ acts freely on a finite dimensional CW-complex X which is homotopy equivalent to a product of spheres $(S^1)^l \times S^{n_{l+1}} \times \cdots \times S^{n_k}$, where each $n_i \geq 2$ and $1 \leq l < k$. Since $S^{n_{l+1}} \times \cdots \times S^{n_k}$ is simply connected, it follows from Theorem 3.12 that $r - \mathrm{rk}_p(\pi_1(F\backslash X)) \leq \dim((S^1)^l) = l$. Now consider the following:

$$\begin{array}{c} \widetilde{X} \simeq \mathbb{R}^l \times S^{n_{l+1}} \times \cdots \times S^{n_k} \\ \downarrow \\ X \simeq (S^1)^l \times S^{n_{l+1}} \times \cdots \times S^{n_k} \\ \downarrow \\ F\backslash X \end{array}$$

Note that $\mathit{\Pi} = \pi_1(F\backslash X)$ acts freely on $\widetilde{X} \simeq S^{n_{l+1}} \times \cdots \times S^{n_k}$. Since $\mathit{\Pi}$ fits into the extension $0 \to \mathbb{Z}^l \to \mathit{\Pi} \to F \to 1$, it is virtually torsion-free. Thus $\mathrm{rk}_p(\mathit{\Pi})$ is well defined. Consider a finite subgroup H of $\mathit{\Pi}$. Since H acts freely on $\widetilde{X} \simeq S^{n_{l+1}} \times \cdots \times S^{n_k}$, we have $\mathrm{rk}_p(H) \leq k - l$ by the assumption. Thus $\mathrm{rk}_p(\mathit{\Pi}) \leq k - l$. Hence $r \leq l + k - l = k$. □

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School of Mathematics, Korea Institute for Advanced Study (KIAS), 207-43 Cheongryangri 2-dong, Dongdaemun-gu, Seoul 130-722, KOREA
E-mail address: jhjo@kias.re.kr